

1-sided test of Significance for p ("less than" variety)

Hypotheses: $H_o : p = p_0$ vs $H_a : p < p_0$

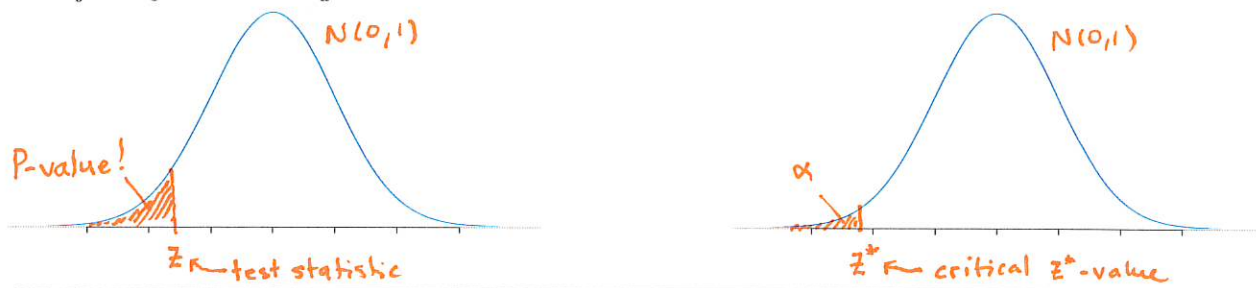
Summary Statistics: n = sample size; \hat{p} = sample proportion.

Test statistic:
$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

P-value: The proportion of the $N(0,1)$ dist'n less than the test stat z . Found in R with `pnorm(z)`

Critical z-score associated to significance level α : The critical z-score is $z^* = \text{qnorm}(\alpha)$.

That is, z^* is the z-score below which one finds area α under the bell curve. The Rejection Region (RR) is the set of z-scores less than z^* . If our test statistic is less than z^* we reject H_o in favor of H_a . Alternatively, we reject H_o in favor of H_a if the P-value of our test is less than α .



Example ("less than" alternative): Do fewer than 1/2 of all voters favor a particular issue? Conduct a test of significance at the $\alpha = .05$ level.

Hypotheses: $H_o : p = 0.5$ vs $H_a : p < 0.5$.

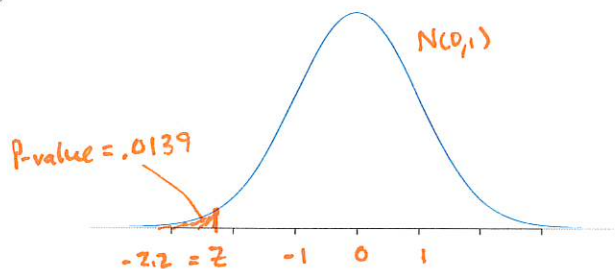
Sample: In an independent sample of 100 voters, 39 say "yes".

This means that $n = 100$, and $\hat{p} = 39/100 = 0.39$.

Test Statistic:
$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.39 - 0.50}{\sqrt{\frac{0.5 \cdot (1-0.5)}{100}}} = \frac{-0.11}{0.05} = -2.2.$$

P-value of the test: `pnorm(-2.2) = .0139`.

Interpretation of the P-value: If 1/2 of all voters favor the issue (in other words, if the null hypothesis is true), the probability of obtaining a sample proportion less than or equal to $\hat{p} = .39$ in a sample of 100 voters, is about 0.0139, the area shaded below.



Conclusion based on the P-value: Since the P-value is less than $\alpha = .05$, we have enough evidence to reject H_o in favor of the alternative, and we conclude from the sample and our analysis that fewer than 1/2 of all voters favor this issue.

Conclusion based on critical z^* -value: Testing at the $\alpha = .05$ level with a "less than" alternative hypothesis, the critical z^* -value is $z^* = \text{qnorm}(.05) = -1.645$. Since our test statistic of $z = -2.2$ is less than $z^* = -1.645$ we reject H_o in favor of H_a at the 5% level.

1-sided test of Significance for p ("greater than" variety)

Hypotheses: $H_o : p = p_0$ vs $H_a : p > p_0$

Summary Statistics: $n =$ sample size; $\hat{p} =$ sample proportion.

Test statistic:
$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

P-value: The proportion of the $N(0, 1)$ dist'n greater than the test stat z . Found in R with $1 - \text{pnorm}(z)$

Critical z-score associated to significance level α : The critical z-score is $z^* = \text{qnorm}(1 - \alpha)$.

That is, z^* is the z-score above which one finds area α under the bell curve. The Rejection Region (RR) is the set of z-scores greater than z^* . If our test statistic is greater than z^* we reject H_o in favor of H_a . Alternatively, we reject H_o in favor of H_a if the P-value of our test is less than α .



Example ("greater than" alternative): Do more than 30% of all high school students participate in music or theatre programs before graduation? Conduct a test of significance at the $\alpha = .01$ level.

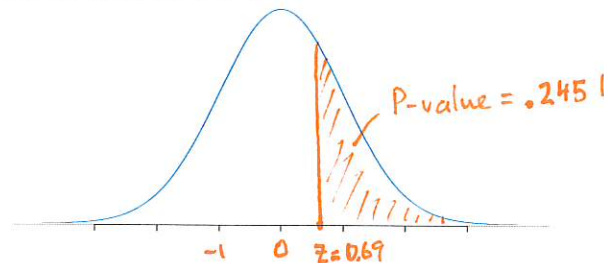
Hypotheses: $H_o : p = 0.3$ vs $H_a : p > 0.3$.

Sample: In an independent sample of 250 students, 80 say "yes".
This means that $n = 250$, and $\hat{p} = 80/250 = 0.32$.

Test Statistic:
$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.32 - 0.30}{\sqrt{\frac{0.3 \cdot (1-0.3)}{250}}} = \frac{0.02}{0.05} = 0.69.$$

P-value of the test: $1 - \text{pnorm}(0.69) = .2451$.

Interpretation of the P-value: If 30% of all high school students participate in music or theatre before graduation, the probability of obtaining a sample proportion greater than or equal to $\hat{p} = .32$ in a sample of 250 students, is about 0.2451 the area shaded below.



Conclusion based on the P-value: Since the P-value is not less than $\alpha = .01$, we do not have enough evidence to reject H_o in favor of the alternative. These data do not provide statistically significant evidence that more than 30% of high school students participate in music or theatre programs prior to graduation.

Conclusion based on critical z^* -value Testing at the $\alpha = .01$ level with a "greater than" alternative hypothesis, the critical z^* -value is $z^* = \text{qnorm}(.99) = 2.326$. Since our test statistic of $z = 0.69$ is not greater than $z^* = 2.326$ we fail to reject H_o in favor of H_a at the 1% level.

2-sided test of Significance for p ("not equal to" variety)

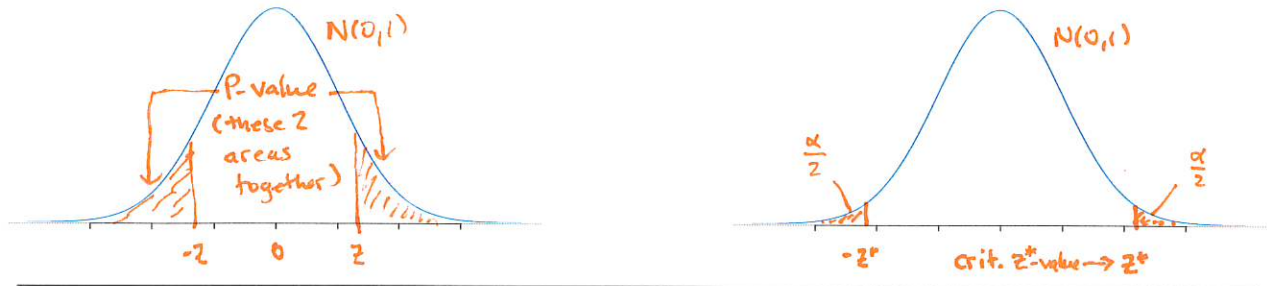
Hypotheses: $H_o : p = p_0$ vs $H_a : p \neq p_0$

Summary Statistics: n = sample size; \hat{p} = sample proportion.

Test statistic:
$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

P-value: The proportion of the $N(0, 1)$ dist'n further away from 0 than the test stat z . Found in R with $2 \cdot \text{pnorm}(-\text{abs}(z))$.

Critical z-score associated to significance level α : The critical z-score is $z^* = |\text{qnorm}(\alpha/2)|$. That is, z^* is the z-score above which one finds area $\alpha/2$ under the bell curve. Then, a total area of α is found above z^* and below $-z^*$. If our test statistic z is such that $|z| > z^*$ then we reject H_o in favor of H_a . Alternatively, we reject H_o in favor of H_a if the P-value of our test is less than α .



Example ("not equal to" alternative): Do 2/3 of Americans support a certain constitutional amendment? Test at the $\alpha = .05$ level.

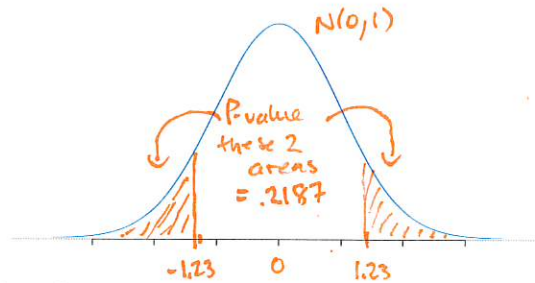
Hypotheses: We choose the two-sided alternative: $H_o : p = 2/3$ vs $H_a : p \neq 2/3$.

Sample: In an independent sample of 1000 Americans, 685 say "yes". This means that $n = 1000$, and $\hat{p} = 0.685$.

Test Statistic:
$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.685 - 2/3}{\sqrt{\frac{2/3 \cdot (1-2/3)}{1000}}} = \frac{0.02}{0.05} = 1.23.$$

P-value of the test: $2 \cdot \text{pnorm}(-1.23) = .2187$.

Interpretation of the P-value: If 2/3 of all Americans supported the amendment, the probability of obtaining a sample proportion further away from 2/3 than $\hat{p} = .685$ in a sample of size 1000 is about 0.2187 the area shaded below.



Conclusion based on the P-value: Since the P-value is not less than $\alpha = .05$, we do not have enough evidence to reject H_o in favor of the alternative. These data do not provide statistically significant evidence that the percentage of Americans supporting this amendment is different than 2/3.

Conclusion based on critical z^* -value Testing at the $\alpha = .05$ level with a "not equal to" alternative hypothesis, the critical z^* -value is $z^* = |\text{qnorm}(.025)| = 1.96$. Since the absolute value of our test statistic of $z = 1.23$ is not greater than $z^* = 1.96$ we fail to reject H_o in favor of H_a at the 5% level.