Section 8.4
Inference for Linear Regression
Based on content in OpenIntro Stats, 4th Ed

Gear up for Inference

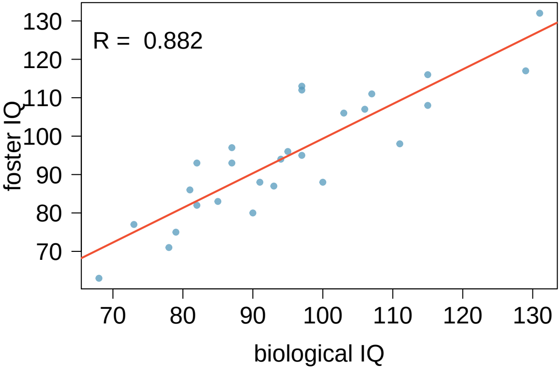
- Inference in this class has been about this: Make a decision about a parameter based on a test statistic generated from good data.
- Inference for linear regression is about this too.
- We assume two variables x and y have a linear association plus some noise:

$$y = \beta_0 + \beta_1 x + \epsilon.$$

- In this theoretical description, β_0 and β_1 are parameters, a sort of theoretical y-intercept (β_0) and theoretical slope (β_1) describing the association.
- We make a decision about β_1 by gathering data, generating a test statistic, and analyzing it (finding a p-value).

Nature or nurture?

In 1966 Cyril Burt published a paper called "The genetic determination of differences in intelligence: A study of monozygotic twins reared together and apart". The data consist of IQ scores for [an assumed random sample of] 27 identical twins, one raised by foster parents, the other by the biological parents.



Which of the following is false?

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Coefficients:
                Estimate Std. Error t value Pr(>|t|)
(Intercept)      9.20760    9.29990   0.990    0.332
bioIQ            0.90144    0.09633   9.358  1.2e-09

Residual standard error: 7.729 on 25 degrees of freedom
Multiple R-squared: 0.7779,Adjusted R-squared: 0.769
F-statistic: 87.56 on 1 and 25 DF,  p-value: 1.204e-09
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(a) Additional 10 points in the biological twin's IQ is associated with additional 9 points in the foster twin's IQ, on average.

(b) Roughly 78% of the foster twins' IQs can be accurately predicted by the model.

(c) The linear model is $\widehat{fosterIQ} = 9.2 + 0.9 \times bioIQ$.

(d) Foster twins with IQs higher than average IQs tend to have biological twins with higher than average IQs as well.

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Testing for the slope

Assuming that these 27 twins comprise a representative sample of all twins separated at birth, we would like to test if these data provide convincing evidence that the IQ of the biological twin is a significant predictor of IQ of the foster twin. What are the appropriate hypotheses?

(a) $H_0 : b_0 = 0$; $H_A : b_0 \neq 0$
(b) $H_0 : \beta_0 = 0$; $H_A : \beta_0 \neq 0$
(c) $H_0 : b_1 = 0$; $H_A : b_1 \neq 0$
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Testing for the slope (cont.)

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	9.2076	9.2999	0.99	0.3316
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- Point estimate $= b_1$, the observed slope.
- SE_{b_1} is the standard error associated with the slope (given in the table!)
- Degrees of freedom associated with the slope is $df = n - 2$, where n is the sample size.
  (We lose 1 degree of freedom for each parameter we estimate, and in simple linear regression we estimate 2 parameters, β_0 and β_1.)

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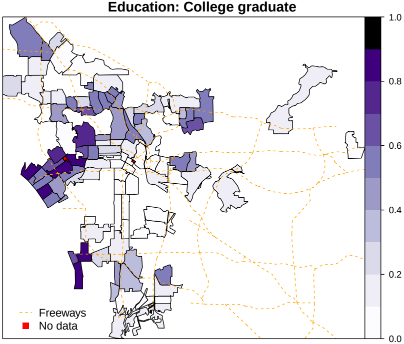
$$\begin{aligned} T &= \frac{0.9014 - 0}{0.0963} = 9.36 \\ df &= 27 - 2 = 25 \\ p - value &= P(|T| > 9.36) < 0.01 \end{aligned}$$

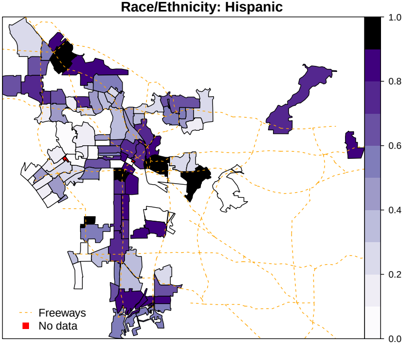
In fact, p-value is:

```
> 2*(1-pt(9.36,25))
[1] 1.197331e-09
```

% College graduate vs. % Hispanic in LA

Q: *What can you say about the relationship between % college graduate and % Hispanic in a sample of 100 zip code areas in LA?*





% College educated vs. % Hispanic in LA - another look

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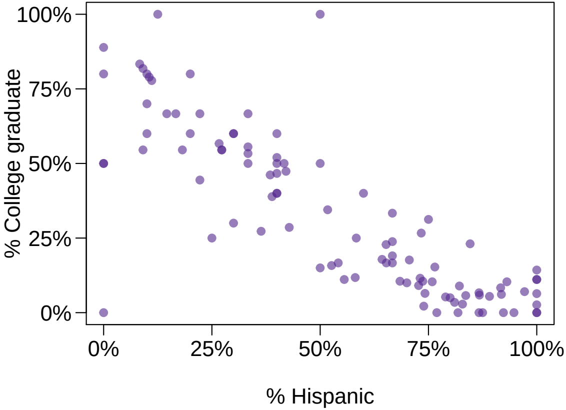

% College educated vs. % Hispanic in LA - linear model

Which of the below is the best interpretation of the slope?

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	0.7290	0.0308	23.68	0.0000
%Hispanic	-0.7527	0.0501	-15.01	0.0000

(a) A 1% increase in Hispanic residents in a zip code area in LA is associated with a 75% decrease in % of college grads.

(b) A 1% increase in Hispanic residents in a zip code area in LA is associated with a 0.75% decrease in % of college grads.

(c) An additional 1% of Hispanic residents decreases the % of college graduates in a zip code area in LA by 0.75%.

(d) In zip code areas with no Hispanic residents, % of college graduates is expected to be 75%.

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% College educated vs. % Hispanic in LA - linear model

Q: *Do these data provide convincing evidence that there is a statistically significant relationship between % Hispanic and % college graduates in zip code areas in LA?*

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Q: How reliable is this p-value if these zip code areas are not randomly selected? *Not very...*

Confidence interval for the slope

Remember that a confidence interval is calculated as *point estimate* $\pm$ *ME* and the degrees of freedom associated with the slope in a simple linear regression is $n - 2$. Which of the below is the correct 95% confidence interval for the slope parameter? Note that the model is based on observations from 27 twins.

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(a) $9.2076 \pm 1.65 \times 9.2999$

(b) $0.9014 \pm 2.06 \times 0.0963$

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$$
\begin{aligned}
n &= 27 \qquad df = 27 - 2 = 25 \\
95\% : \ t^\star_{25} &= 2.06 \\
0.9014 &\pm 2.06 \times 0.0963 \\
(0.7 &, \ 1.1)
\end{aligned}
$$

Recap

- Inference for the slope for a single-predictor linear regression model:
  - Hypothesis test:

$$T = \frac{b_1 - null\ value}{SE_{b_1}} \qquad df = n - 2$$

  - Confidence interval:

$$b_1 \pm t^{\star}_{df=n-2} SE_{b_1}$$

- The null value is often 0 since we are usually checking for *any* relationship between the explanatory and the response variable.
- The regression output gives b_1, SE_{b_1}, and *two-tailed* p-value for the t-test for the slope where the null value is 0.
- We rarely do inference on the intercept, so we'll be focusing on the estimates and inference for the slope.

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- If you have a sample that is non-random (biased), inference on the results will be unreliable.
- The ultimate goal is to have independent observations.