

Worksheet: Hypothesis Tests for a Population Proportion

Hypothesis Tests for a Population Proportion

Suppose we want to test a claim about a population proportion, say whether $p = p_o$ for some particular value p_o .

We test the null hypothesis $H_o : p = p_o$ against one alternative, either $H_a : p < p_o$, or $H_a : p > p_o$, or $H_a : p \neq p_o$, depending on the research question.

The **test statistic**, computed assuming H_o is true (and the conditions of the CLT are met), and based on our sample of size n in which we found sample proportion \hat{p} is:

$$z = \frac{\hat{p} - p_o}{\sqrt{p_o(1 - p_o)/n}}$$

The **P-value** of the test is the probability of obtaining a test statistic more extreme than the one we got.

The smaller the P-value, the smaller the probability of obtaining our observed sample proportion if the null is true. So, the smaller the P-value, the greater the evidence against the null hypothesis.

Often a test has a significance level α associated with it, which is used as follows:

- If the P-value is less than α we reject H_o in favor of H_a .
- If the P-value is greater than or equal to α we fail to reject H_o in favor of H_a .

1. In each scenario below conduct the appropriate test of significance: Calculate the test statistic, determine the P-value, and state your conclusion.

(a) $H_o : p = 0.4$ vs $H_a : p < 0.4$; Data: $n = 200$, $\hat{p} = 88/200$. Significance level $\alpha = .05$

(b) $H_o : p = 0.5$ vs $H_a : p \neq 0.5$; Data: $n = 200$, $\hat{p} = 80/200$. Significance level $\alpha = .01$

2. Do more than $1/2$ of *all* voters favor a particular issue? To address this question we gather an independent sample of size $n = 250$ and find 132 favor the issue. Conduct a test of significance at the $\alpha = .05$ level. (i) Define p , the parameter of interest. (ii) State hypotheses, (iii) calculate the test statistic from the sample, (iv) determine the P-value and (v) state your conclusion in words.

3. If you flip a quarter 100 times and you flip only 39 heads, do you have statistically significant evidence at the $\alpha = .01$ level that the coin is not fair? Conduct a test of significance to decide, assuming your sample is a nice, independent sample.