MATH 140 **Name** _____ **Date:**

Worksheet: Random Phones

The Scene: After class I find 4 phones in the classroom, and at the start of the next class I randomly return these phones to the 4 students who misplaced them. What is the probability that I return all four phones to the correct owner?

Strategy: We estimate this probability by simulating many instances of this event, recording how many students receive their own phone back each time.

First approach: **Simulation using cards**

We can simulate the returning of the four phones by using cards, as follows. Take 4 cards (one of each suit: hearts, clubs, spades, diamonds) and a sheet of scratch paper. Divide the paper into four areas and label each area with a suit. Then shuffle the cards and randomly place them onto the four areas of the paper. If the suit on the card matches the suit of the area, then we have a match (and a phone returned to the right student!). How many matches do you get?

1. Simulate the returning of the phones 10 times. In the table below, record how many phones found their owner in each of the 10 trials.

trial	1	2	3	4	5	6	7	8	9	10
# of matches										

2. Record the counts (aka frequencies) for your own results in the row of the table below marked 'your freq'. We will also pool the entire class data, and then you can record the class results in the other row.

matches	0	1	2	3	4
your freq			.		
class freq					

3. Determine the **relative frequency** for each outcome for both your own results, and the pooled class results. The relative frequency is the frequency divided by the total number of trials. Note: The sum of all the relative frequencies must add up to 1.

matches	0	1	2	3	4
your rel freq			.		
class rel freq					

4. Explain briefly why exactly 3 matches is not possible.

5. Based on the simulation results of the class, what outcome is most likely, 0, 1, 2, or 4 matches?

6. Based on the simulation results of the class, what is your empirical estimate of the probability of at least one match?

7. Is it impossible to get four matches? Would you call it rare? Unlikely?

Second Approach: Simulation using RStudio

FYI, the following code will conduct this simulation 10,000 times very quickly!

```
phones=c("d","h","s","c")
trials=10000
matches = c()
for (i in 1:trials){
  matches = c(matches,sum(phones == sample(phones)))
  }
table(matches)/trials #relative frequency table
```

The results of one such simulation of 10,000 trials is given in the table below.

matches	0	1	2	3	4
rel freq	.3709	.3356	.2530	0	.0405

8. Based on the simulation results table above, are your answers to questions 5-7 still reasonable? Explain.

Third Approach: Write down all the possible outcomes

9. The table below lists all 24 possible arrangements of the numbers 1,2,3,4. Each arrangement can be thought of as "returning" the four numbers to the spots 1 through 4. We have a match if a number is in its 'home' spot (i.e., 1 appears in the first spot, 2 is in the second spot, 3 in the third, or 4 in the fourth). For instance, "1243" has two matches, since 1 is in the first spot and 2 is in the second (but 3 and 4 are not 'home'); meanwhile, "2413" has 0 matches - no number occurs in its correct spot. Record how many matches each arrangement has.

Arrangement	Matches	Arrangement	Matches	Arrangement	Matches
1234	4	2314		3412	
1243	2	2341		3421	
1324		2413	0	4123	
1342		2431		4132	1
1423		3124		4213	
1432		3142		4231	
2134		3214		4312	
2143		3241		4321	

10. Assuming each of the 24 outcomes is equally likely, use the previous problem to determine the probability model for the number of matches X if the four phones are returned at random. Write your answers below as fractions, not decimals.

x	0	1	2	3	4
$p(x)$				0	

11. How close are the relative frequencies from the R simulation to the theoretical probabilities determined in the previous problem? What is the name of the Law that assures us they will be close, given we did so many trials in the R simulation?