MATH 140 **Name** ______________________ **Date:**

Worksheet: The Central Limit Theorem for Sample Proportions

The Central Limit Theorem for Sample Proportions

When we collect a sufficiently large sample of n independent observations from a population with population proportion of p, the sampling distribution of $\hat{p}$ will be nearly normal:

$$N\left(p, \sqrt{\frac{p(1-p)}{n}}\right).$$

The sample size is typically considered sufficiently large when the success-failure condition has been met: $np \geq 10$ and $n(1-p) \geq 10$, but n does not exceed 10% of the population.

1. Suppose a jar is full of orange and green marbles. To be precise, the jar contains 800 orange marbles and 3200 green marbles, although the marbles are all mixed up, and you select a random sample of $n = 100$ marbles from the jar. We will be interested in $\hat{p}$, the proportion of orange marbles in your sample.

   (a) Determine p, the proportion of orange marbles in the entire jar.

   (b) Compute np, which represents your best guess at how many orange marbles you would expect in your sample.

   (c) Compute $n(1-p)$, which represents your best guess at how many green marbles you would expect in your sample.

   (d) If both of your answers above are at least 10, then the success-failure condition of the CLT for proportions has been met. Has this condition been met?

   (e) According to the CLT, the sampling distribution for $\hat{p}$, the sample proportion of orange marbles in a random sample of $n = 100$ marbles, is normally distributed. What is the mean, and what is the standard deviation of this distribution?

   (f) Now, use the CLT to estimate the probability that in your sample of $n = 100$ marbles, your sample proportion of orange marbles, $\hat{p}$, is greater than or equal to 0.4. That is, estimate $P(\hat{p} \geq .4)$.

2. Suppose that a widget manufacturer finds that one-fifth of the widgets coming off its assembly line are defective. We view $p = 1/5$ as a parameter, it's the population proportion of all widgets that will be defective. Suppose one inspects a random sample of $n = 200$ widgets coming off the assembly line. Check that the success-failure condition has been met here, and then sse the CLT to estimate the probability that more than 60 of them are defective.

3. Now suppose engineers for the widget manufacturer make modifications to the production process that they believe reduces the probability of a given widget being defective. After the modifications are made, a new sample of $n = 200$ widgets is obtained, and they find that only 30 defective. If the modifications had no effect on the production process, and p is still $1/5$, use the CLT to estimate the probability that in a random sample of 200 widgets 30 or fewer are defective.

4. Based on your answer to the previous question, what do you think? Do you think the modifications really helped reduce the chance of producing a defective widget, or could the sample we saw happen fairly easily if $p = 1/5$ still?