MATH 140 **Name** Key **Date:**

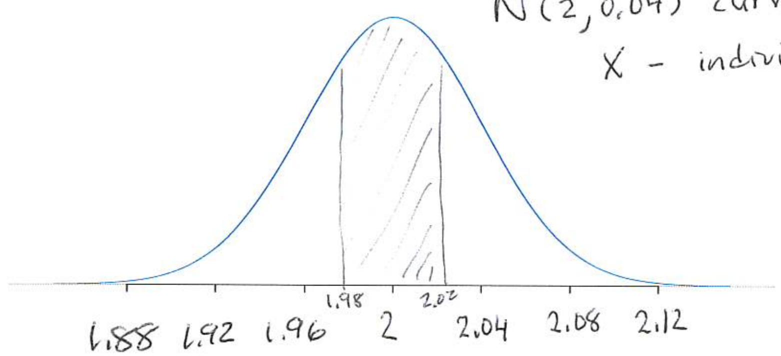
Worksheet: The Central Limit Theorem for Sample Means

> **The Central Limit Theorem for Sample Means**
>
> When we collect a sufficiently large sample of n independent observations from a population with mean μ and standard deviation σ, the sampling distribution of $\overline{x}$ will be nearly normal with
>
> $$\text{Mean} = \mu \text{ and Standard Error (SE)} = \frac{\sigma}{\sqrt{n}}.$$

1. Suppose the actual weight of a certain candy bar, whose advertised weight is 2 ounces, varies according to a normal distribution with mean $\mu = 2$ ounces and standard deviation $\sigma = 0.04$ ounces.

   (a) Label the tick marks on the bell curve below so that it represents the distribution of weights of this type of candy bar.

   $N(2, 0.04)$ curve for weight of X - individual candy bars



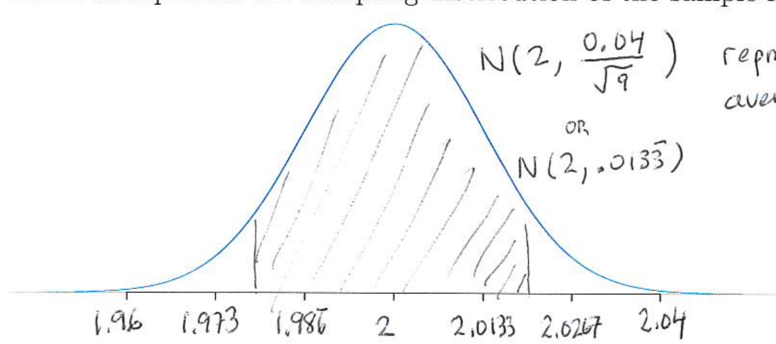
   (b) What is the probability that an individual candy bar will weight between 1.98 and 2.02 ounces? Sketch the probability of interest as an area in the density curve above, convert the question to z-scores, and use a probability table or R to find your answer.

   $$Z_{low} = \frac{1.98-2}{0.04} = -0.5 \qquad P(1.98 < X < 2.02) = P(-0.5 < Z < 0.5)$$

   $$Z_{high} = \frac{2.02-2}{0.04} = 0.5 \qquad = \text{pnorm}(0.5) - \text{pnorm}(-0.5) = .3829$$

   (c) Suppose that you plan to take a simple random sample of 9 candy bars and calculate the sample mean weight, $\overline{x}$. What does the Central Limit Theorem say about how these sample means would vary from sample to sample? Label the tick marks on the bell curve below to represent the sampling distribution of the sample mean in this problem.

   $N(2, \frac{0.04}{\sqrt{9}})$ OR $N(2, .013\overline{3})$ represents the distribution for the average weight $\overline{X}$ of 9 candy bars



   (d) Shade in the region on the bell curve above that corresponds to the probability that the sample mean weight of these nine candy bars will fall between 1.98 and 2.02 ounces.

Determine this probability by converting to z-scores and using R, a table, or a calculator. Is this probability greater or less than your answer to (b)?

$$z_{low} = \frac{1.98-2}{.013\bar{3}} = -1.5 \qquad \text{and} \quad P(1.98 < \bar{X} < 2.02) = P(-1.5 < Z < 1.5)$$

$$= \text{pnorm}(1.5) - \text{pnorm}(-1.5)$$

$$z_{high} = \frac{2.02-2}{.0133} = +1.5 \qquad = 0.8664$$

Much more likely for the average weight of 9 bars to be between 1.98 & 2.02 ounces, than any single candy bar.

(e) Now, consider the more realistic assumption that you do not know the value of the population mean weight μ of the candy bars. What can you say about the probability that a sample of size $n = 25$ would result in a sample mean weight $\bar{x}$ within ± 0.01 of the actual population mean μ?

For $\bar{X}$ with $n=25$, sampling dist'n is $N(\mu, \frac{.04}{\sqrt{25}})$ or $N(\mu, .008)$

Then we want $P(\mu - 0.01 < \bar{x} < \mu + 0.01)$

$$z_{low} = \frac{(\mu - 0.01) - \mu}{0.008} = -1.25$$

$$z_{high} = \frac{(\mu + 0.01) - \mu}{0.008} = 1.25$$

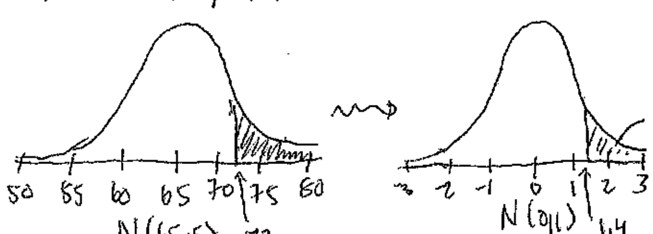
So Probability of sample mean $\bar{x}$ within ± 0.01 of μ is

pnorm(1.25) − pnorm(−1.25) = .7887

2. The weight of the eggs produced by a certain breed of hen is normally distributed with mean 65 grams and standard deviation 5 grams.

(a) Determine the probability that a single egg, chosen at random, weighs more than 72 grams.

$X \sim N(65, 5)$. $\quad Pr(X > 72) = Pr\left(Z > \frac{72-65}{5}\right) = Pr(Z > 1.4)$



$Pr(Z > 1.4) = 1 - \text{pnorm}(1.4) = .0808$

(b) Treat a carton of 12 eggs as a SRS of size 12 from the population. What is the probability that the weight of a carton falls between 750 and 825 grams? Hint: Convert this question to a question about the sample mean of 12 eggs.

First rework in terms of average. Total weight of 12 between 750 & 825 grams means avg weight of 12 between 62.5 and 68.75

Also $\bar{X} \sim N(65, \frac{5}{\sqrt{12}})$

$$z_{low} = \frac{62.5 - 65}{5/\sqrt{12}} = -1.73$$

$$z_{high} = \frac{68.75 - 65}{5/\sqrt{12}} = 2.60$$

$Pr(62.5 < \bar{X} < 68.75) = \text{pnorm}(2.60) - \text{pnorm}(-1.73)$

$= .9535$