

Confidence Interval for a Population Proportion

When we collect a sufficiently large sample of n independent observations from a population with unknown population proportion p , the CLT provides us with a likelihood that our sample proportion \hat{p} is close to p , and we can use \hat{p} to construct a **level C confidence interval for p** :

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}.$$

About z^*

The value z^* depends on the confidence level C . For instance,
for a 95% confidence interval, $z^* = 1.96$, and
for a 90% confidence interval $z^* = 1.645$.

In general, z^* is the value in $N(0, 1)$ such that the proportion of $N(0, 1)$ between $-z^*$ and z^* is C , and $z^* = \text{qnorm}(C + (1 - C)/2)$.

Example: For a 95% confidence interval, $C = .95$, and
 $z^* = \text{qnorm}(.95 + .025) = \text{qnorm}(.975) = 1.96$.

1. Suppose we want to estimate the proportion of voters in a large city who support a particular candidate for mayor. In an independent sample of $n = 200$ voters it turns out that 108 of the 200 voters support this candidate. Determine a 95% confidence interval for the true proportion of voters in the city who support this candidate.
2. Suppose a second independent sample, taken closer to the election, finds that 104 of the 190 people asked support the candidate. Determine a 98% confidence interval for the true proportion of voters in the city who support this candidate.
3. Interpret the meaning of the interval you calculated in Q2. Based on this interval, how confident are you that the candidate will win the election?